Bailouts, Bank Runs, and Signaling

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Abstract

During the recent financial crisis, there were many bank runs and government bailouts. In many cases, bailouts did not prevent but trigger bank runs. This paper develops a global game bank run model to study the optimal government bailout policy. The endogenous information generated by government bailout leads to multiple equilibria. The government will be trapped in a position where the effectiveness of bailout is dictated by self-fulfilling expectations.
1 Introduction

There were widespread bailouts and bank runs during the 2008-2009 financial crisis. Even for large banks\(^1\) we saw runs on Bear Stearns, Lehman Brothers, Washington Mutual, and Wachovia. Among them, Washington Mutual is the largest bank failure in the US financial history, and Lehman Brothers bankruptcy is the largest in the US history. 165 banks failed during the years of 2008 and 2009, compared to only 11 bank failures in the five years before 2008.

We also saw the largest government interventions in the US financial history. Government interventions are in the form of various bailouts. Even though the stated goal of this intervention is to “restore confidence to our financial system”, several bank runs happened right after government bailout announcement. This is not rare in other countries. For example, in 2007, right after Bank of England announced they would rescue Northern Rock, which was the fifth largest bank in the UK by mortgage asset, on the next day, depositors queued in front of Northern Rock to withdraw their money, exactly like the traditional bank runs. For details, please read Shin (2009). Northern Rock was the first bank in 150 years in UK to suffer a bank run, which triggers economist’s interest in studying bank runs. For the US financial market, one of the most prominent government interventions was TARP (Troubled Asset Relief Program), which was proposed by Henry Paulson, the then US Treasury Secretary. The 700 billion dollar TARP plan mostly injected capital to US banks in various ways. Banking is about confidence, while stock market is a good approximate of investor confidence. Even after the proposal was passed by the congress, the US stock market experienced the worst week ever. Bear Stearns is a case study.

In his description of bank runs during the 2007-2009 crisis, Markus K. Brunnermeier (Brunnermeier (2009)) wrote

“...on March 11 [2008], ... the Federal Reserve announced its $200 billion Term Securities Lending Facility. This program allowed investment banks to swap agency

\(^1\)In this paper, we call all the financial institutions as banks. For the shadow banking system, maturity mismatch problem still applies.
and other mortgage-related bonds for Treasury bonds for up to 28 days.... Naturally, they (market participants) pointed to the smallest, most leveraged investment bank with large mortgage exposure: Bear Stearns......Bear’s liquidity situation worsened dramatically the next day as it was suddenly unable to secure funding on the repo market.”

These phenomena urge our study on bailouts and bank runs. This paper develops a global game signaling model based on Diamond and Dybvig (1983), Morris and Shin (2001), and Angeletos, Hellwig, and Pavan (2006). There are three periods, 0, 1, 2. The setting is similar with Diamond and Dybvig (1983). Banks have the long term productive technology which can transform 1 unit of investment in period 0 to \( R > 1 \) units in period 2, while investors don’t have. Investors put their money into a bank, and they can withdraw their money at any time they want. Banks face the liquidation risk where long term investments are liquidated. That more investors withdraw early leads to more liquidations, which leads to a lower final total return. So there exists coordination failures among investors. In the framework of Diamond and Dybvig (1983), there are multiple equilibria where either everyone runs on the bank in period 1 or no one runs in period 1. Here, we employ the global game technique developed by Morris and Shin (2001) assuming that investors get a private signal of final return to obtain a unique equilibrium where investors whose signals are below some threshold run on the bank in period 1, otherwise they wait until period 2. We add a government sector to study government bailout decisions and bank runs. Government objective function is decreasing in liquidation cost, i.e., the number of investors who run on the bank, and bailout cost. A larger government bailout has the tradeoff of reducing the number of early withdrawals and increasing the bailout cost. However, in our model, bailout also reveals government’s signal of the final return. Optimal government bailout policy should take the signaling role of bailouts into consideration. Our model then has multiple equilibria, which can be divided into two parts. The first one is pooling equilibrium, where government commits never to bail out any banks, since investors’ belief does not depend on the bailout amount. The second part is semi-separating equilibria, where there are a continuum of equilibria. Government chooses to bail out the banks whose

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2 More empirical results on bailouts and bank runs can be found in Wang (2011).

3 Running on the bank and withdrawing money at period 1 are used interchangeably with the same meaning.
fundamental or return is in the middle range. If bank return is sufficiently high, then there will be fewer early withdrawals. So there is no need for the government to bail out the bank. If the bank is too bad, then the government opts not to bail out. Because the bailout cost will be much higher.

Section 2 is the main part of this paper which presents the model and its result. Section 3 concludes.

2 The Model

The model follows from Morris and Shin (2001), which is a variation of Diamond and Dybvig (1983). There are three periods, \( t = \{0, 1, 2\} \), and a continuum of agents, each endowed with one unit of endowment. Agents are the same at period 0. Agents do not know their type until period 1. At period 1, a measure \( \lambda \) of the agents become impatient agents, and a measure 1 of the agents become patient agents. Impatient agents care only about the first period consumption, while patient agents can have consumption at period 1 and 2 with perfect substitutes. Agents become impatient with probability \( \frac{\lambda}{1+\lambda} \) and become patient with probability \( \frac{1}{1+\lambda} \). Agents have log utility functions. Impatient agents’ utility function takes the following form,

\[
u(c_1) = \log c_1\]

where \( c_1 \) is first period consumption. Patient agents have the following utility function

\[
u(c_1, c_2) = \log(c_1 + c_2)\]

where \( c_2 \) is second period consumption.

Agents have the choice of storing his own endowment or invest it in a bank. Storing by himself will give him one unit back either at period 1 or at period 2. Only the bank has the productive technology of transforming one unit of endowment at period 0 into \( R \) units, where
$R > 1$, only after two periods. The bank also has the option to just store the investment to get
one unit back later, same as the storage technology possessed by the agents. If $l$ proportion
of the investment in the technology is withdrawn at period 1, the rate of return is reduced to
$R \cdot e^{-l}$. For simplicity, I assume $r = \log R$. If there is liquidation, the rate of return can be
reduced to $e^{r-l}$. I assume $0 < r < 1$. $l$ is a measure of the cost of liquidation.

2.1 Optimal Contract

The optimal contract is mainly to solve the following maximization problem.

$$\max_{\{c_1, c_2\}} \frac{\lambda}{1+\lambda} u(c_1) + \frac{1}{1+\lambda} u(c_2)$$

s.t.

$$\lambda c_1 + \frac{c_2}{R} \leq 1 + \lambda$$

$$u(c_1) \leq u(c_2)$$

The objective function is the total welfare of agents. With probability $\frac{\lambda}{1+\lambda}$, the agent becomes
impatient agent who cares only about the first period consumption. With probability $\frac{1}{1+\lambda}$, the agent becomes patient. With the incentive compatibility constraint $u(c_1) \leq u(c_2)$, the agent’s total welfare is $\frac{1}{1+\lambda} u(c_2)$. The incentive compatibility constraint states that patient agents choose to not run on the bank. The other constraint is the total resource constraint. Since there is liquidation cost, the bank will hold $\lambda c_1$ amount of investment in cash and invest $\frac{c_2}{R}$ into the productive technology. The total investment cannot exceed the total resources $1 + \lambda$. Excluding the incentive compatibility constraint, we can get $c_1 = 1$ and $c_2 = R$. These two values satisfy the incentive compatibility constraint. Therefore, $c_1 = 1$ and $c_2 = R$ is the solution to the above problem.

2.2 Government

Now, we add a government sector to the model. Government injects bailout $B \geq 0$ amount of
endowments to the bank at period 1. Assume $b = \log B$. Then, the rate of return is $R \cdot e^{-(l-b)}$
in the sense that the bailout money can be used to offset part of the patient agent withdrawals at period 1.\[4\]

Government has the following payoff function

\[ U = (1 - D)(r + b - l) - C(b) \]

where \(D\) denotes the bank run outcome; \(D = 1\) indicates the bank has gone bankruptcy, otherwise \(D = 0\); \(C\) is the normalized bailout cost, which is an increasing function of \(b\) and \(C(0) = 0\). Both \(D\) and \(b\) are chosen by the government. Patient agent will obtain 1 if he withdraws at period 1 and will obtain \(e^{r+b-l}\) if he withdraws later. Hence, patient agent will withdraw early if \(r + b - l < 0\). Government payoff is a decreasing function of the liquidation cost \(l\). It is assumed in this way because liquidation will generate negative externalities such as fire sales.

### 2.3 Timing and Information

We follow the standard global game literature by assuming information dispersion. Suppose \(r\) is a random variable with normal distribution. It has mean \(\bar{r}\) and precision \(\alpha\), or equivalently variance \(1/\alpha\). To make the model interesting, \(r\) is assumed to be neither too large nor too small. Formally, \(0 < \bar{r} < 1\). Depositor \(i\) observes an imperfect signal \(x_i = r + \varepsilon_i\), where \(\varepsilon_i\) is normally distributed with mean 0 and precision \(\beta\) (i.e., variance \(1/\beta\)). Government here is assumed to be able to obtain the realization of \(r\). \(s(x_i, B)\) is an indicator function which is equal to 1 if patient agent \(i\) who receives signal \(x_i\) and observes bailout amount \(B\), decides to run on the bank, and is equal to 0 otherwise.

The timing of the model can be illustrated as follows. At period 0, investors put their endowments into the bank. At period 1, \(r\) is realized, and government perfectly observes \(r\). Then, each investor receives his private signal about \(r\). The government determines the bailout amount \(B\) by optimizing its payoff function. In the end, investors choose whether to run on the bank or not conditional on \(x_i\) and \(B\).

\(^4\)If \(b < l\), then the return will be \(R \cdot e^{-l(b-l)}\). If \(b \geq l\), then the return will be \(R + B(1-l)\).
2.4 Equilibrium

The model is composed of the signaling game in stage 1 and the coordination game in stage 2. The equilibria are symmetric perfect bayesian.

This section consists of two parts. The first one is the benchmark model where there is no signaling. The second one is the full model. First, government commits never to bail out any bank, i.e. \( B = 0 \). This case is reduced to the basic model of [Morris and Shin (2001)].

**Proposition 1.** Suppose government commits never to bail out any bank, i.e., \( B = 0 \). Provided that \( \gamma \leq 2\pi \), the equilibrium is unique and is such that

\[
 s(x, r) = \begin{cases} 
 1 & \text{if } x < \hat{x} \\
 0 & \text{otherwise} 
\end{cases}
\]

\[
 D(r) = \begin{cases} 
 1 & \text{if } r < \hat{r} \\
 0 & \text{otherwise} 
\end{cases}
\]

where \( \hat{x} \) satisfies

\[
 \frac{\alpha \bar{r} + \beta \hat{x}}{\alpha + \beta} = \Phi\left(\sqrt{\gamma}\left(\frac{\beta(\hat{x} - \bar{r})}{\alpha + \beta}\right)\right)
\]

and \( \hat{r} \) is determined by

\[
 \hat{r} = \Phi\left(\sqrt{\gamma}\left(\frac{\beta(\hat{x} - \bar{r})}{\alpha + \beta}\right)\right)
\]

**Proof.** The distributions in this model are all assumed to be normal. Conditional on \( x_i \), the following is the distribution for \( r \),

\[
r|x_i \sim N(\rho_i, \frac{\alpha + 2\beta}{\beta(\alpha + \beta)})
\]

where \( \rho_i = \frac{\alpha \bar{r} + \beta x_i}{\alpha + \beta} \) and the variance \( \frac{\alpha + 2\beta}{\beta(\alpha + \beta)} \) is from \( \frac{1}{\alpha + \beta} + \frac{1}{\beta} \).
Conditional on $\rho_i$, what’s the probability that investor $i$ expects investor $j$ has posterior lower than him? Since

$$\rho_j < \rho_i \implies \frac{\alpha \bar{r} + \beta x_j}{\alpha + \beta} < \rho_i \implies x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r})$$

then,

$$\text{Prob}(\rho_j < \rho_i | \rho_i) = \text{Prob}(x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) | \rho_i)$$

$$= \Phi\left(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}} \left(\rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) - \rho_i\right)\right)$$

$$= \Phi\left(\sqrt{\gamma(\rho_i - \bar{r})}\right)$$

where $\Phi$ is the cdf of the standard normal distribution and

$$\gamma = \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}.$$

Suppose $\hat{\rho}$ is an equilibrium switching point, which corresponds to the observation of $\hat{x}$. Then, that investor must be indifferent with withdrawing or not, i.e., the expectation of $r - l$ must be equal to 0. Formally,

$$E(r - l | \hat{\rho}) = \hat{\rho} - \Phi(\sqrt{\gamma(\hat{\rho} - \bar{r}))} = 0$$

which is equivalent to $\hat{\rho} = \Phi(\sqrt{\gamma(\hat{\rho} - \bar{r}))}$. Since

$$\hat{\rho} = \frac{\alpha \bar{r} + \beta \hat{x}}{\alpha + \beta},$$

then the switching observation $\hat{x}$ must satisfy the following equation

$$\frac{\alpha \bar{r} + \beta \hat{x}}{\alpha + \beta} = \Phi(\sqrt{\gamma(\frac{\alpha \bar{r} + \beta \hat{x}}{\alpha + \beta} - \bar{r}))}$$

$$= \Phi(\sqrt{\gamma(\frac{\beta(\hat{x} - \bar{r})}{\alpha + \beta})})$$
The bankruptcy threshold can be determined by \( \hat{r} = l = \text{Prob}(\rho < \hat{\rho} | r) = \Phi(\sqrt{\gamma}(\hat{\rho} - \bar{r})) = \Phi(\sqrt{\gamma}(\hat{x} - \bar{x})) \).

From Proposition 1, it can be easily obtained that under this partial equilibrium setting which does not consider government’s objective, any bailout \( b \) will reduce the probability of bank runs, i.e., \( \frac{\partial \hat{\rho}}{\partial b} < 0 \). However, if government’s decision making process is taken into consideration, any bailout amount will convey government’s information to the investors which will affect their behavior. The next proposition states the main result of the paper.

**Proposition 2.** There are multiple equilibria if government bailout is endogenous.

(a) There is an equilibrium in which

\[
\begin{align*}
b(r) &= 0 \text{ for all } r \\
s(x, b) &= \begin{cases} 1 & \text{if } x < \hat{x} \\ 0 & \text{otherwise} \end{cases} \\
D(r) &= \begin{cases} 1 & \text{if } r < \hat{r} \\ 0 & \text{otherwise} \end{cases}
\end{align*}
\]

where \( \hat{x} \) and \( \hat{r} \) are as shown in Proposition 1.

(b) For any \( b^* \in (0, \bar{b}) \), there is an equilibrium in which

\[
\begin{align*}
b(r) &= \begin{cases} b^* & \text{if } r \in [r^*, r^{**}] \\ 0 & \text{otherwise} \end{cases} \\
s(x, b) &= \begin{cases} 1 & \text{if } (x, b) < (x^*, b^*) \\ 0 & \text{otherwise} \end{cases} \\
D(r) &= \begin{cases} 1 & \text{if } r < r^* \\ 0 & \text{otherwise} \end{cases}
\end{align*}
\]
where \( r^*, r^{**}, \) and \( x^* \) are given by

\[
\begin{align*}
    r^* &= C(b^*) - b^* \\
    r^{**} &= r^* + \sigma [\Psi^{-1}(1 - C(b^*)) - \Psi^{-1}(C(b^*))] \\
    x^* &= r^{**} + \sigma \Psi^{-1}(C(b^*))
\end{align*}
\]

**Proof.** (Outline) We first prove the pooling equilibrium which is illustrated in (a) where \( b = 0 \). Then, we switch to the semi-separating equilibria in which \( b > 0 \) for an intermediate interval of \( r \).

(1) When the investors play according to the strategy illustrated in (a), the policy choice \( b \) does not affect the investor’s behavior since investors’ belief does not depend on \( b \). Therefore, it’s optimal for the government to choose \( b = 0 \) to minimize the bailout cost. Consider the investors’ strategy. From government objective function, government chooses to let the bank fail if and only if \( r < \hat{r} \), where \( \hat{r} \) solves \( \hat{r} + b = l(\hat{r}, b) \). When \( b = 0 \), beliefs are pinned down by Bayes rule and satisfy \( \mu(r < \hat{r} | x, b = 0) > \hat{r} \) if and only if \( x < \hat{x} \). For \( b \neq 0 \), out-of-equilibrium beliefs are considered that assign zero measure to types for which the deviation is dominated. Since the policy maker’s payoff is zero for \( r < \hat{r} \) and \( r - l(r, b = 0) > 0 \) for \( r > \hat{r} \), any \( b > \hat{b} \) is dominated in equilibrium for all \( r \), where \( \hat{b} \) is defined as satisfying the equation, \( C(\hat{b}) = \hat{r} + \hat{b} = l(\hat{r}, b = 0) \). A deviation to some \( b' \in (0, \hat{b}) \) is dominated if and only if \( r \notin [r', r''] \), where \( r' \) and \( r'' \) solve \( r' + b' = C(b') = l(r'', b = 0) \): for \( r < r' \), the cost of \( b' \) exceeds the value of keeping the bank alive; for \( r > r'' \), the attack faced in equilibrium is smaller than the bailout cost \( b' \). Since \( \hat{r} \in [r', r''] \), beliefs are constructed by assigning zero measure to \( r \notin [r', r''] \). Given these beliefs, an investor will follow other investors’ strategy in choosing whether to run the bank or not, i.e., all the investors share a same threshold. Since the number of investors who run on the bank is independent of \( b \), the optimal choice will be obviously \( b = 0 \).

(2) For the semi-separating equilibria, if the investors use the strategy in (b), then it is never optimal for the government to have the bailout amount bigger than \( b^* \). Define \( r^* \) and \( r^{**} \) as solving \( r^* + b^* = C(b^*) \) and \( C(b^*) = l(r^{**}, b = 0) \). Government chooses the bailout amount \( b^* \) instead of 0 if and only if the opportunity cost of setting \( b = 0 \), which is \( l(r, b = 0) \),
is higher than the bailout cost $C(b^*)$. The interval $[r^*, r^{**}]$ is the set of types for whom $C(b^*) \leq \min\{r + b, l(r, b = 0)\}$. If $r \in [r^*, r^{**}]$, then government prefers $b^*$ to $b = 0$. Simple algebra leads to the expressions of $r^*$, $r^{**}$, and $x^*$ as described in (b) above. Same proof logic with (1) applies here as follows. Any $r' > r^*$ is dominated in equilibrium for all types. A deviation to $b' \in (0, b^*)$ is not dominated for the intermediate value between $r'$ and $r''$ where $r'$ and $r''$ solve $r' + b' = C(b') = l(r'', b = 0)$ and hence satisfy $0 < r' < r^* \leq r^{**} < r''$. Out-of-equilibrium beliefs can be constructed that sustain the proposed strategy and attach zero probability to types for which a deviation is dominated. Q.E.D.

We can see that in the pooling equilibrium, government commits never to bail out any banks since investors’ bank run decision does not hinge on the bailout amount. For the semi-separating equilibria, there is a continuum of equilibria where investors’ self-fulfilling expectations dictates. Government bails out the bank in the intermediate range of fundamentals, since bailing out too bad banks is too costly and bailing out too good banks is not necessary.

3 Conclusion

In this paper, we develop a model to characterize the optimal bailout government policy when banks face the risk of runs. This paper combines the coordination game and signaling game where signaling appears in the first stage and coordination game in the second stage. The bailout policy is endogenous in the sense that the government takes the investor’s reaction to the bailout policy into consideration. There are two types of equilibria. One is inactive policy equilibrium, where investors are inactive to the policy. So government opts not to bail out any banks. The other one is active policy equilibrium where government bails out the banks whose fundamentals are in the intermediate range. Bailing out too bad banks is too costly and bailing out too good banks is not necessary. Government finds itself trapped in a position where self-fulfilling expectations dictate the equilibrium outcomes.

There are several future extensions to the model. First, if the model is dynamic, investors’ will update their belief about the government type. Second, modern bank features like repo
can be added to the model.

References


